ANOVA: Comparing More Than Two Means

10.3 The Randomized Block Design

1. Researchers want to know which age group has higher average cholesterol levels, so they track cholesterol levels for 4 different subjects. Since the subjects in the study will have different starting cholesterol levels, the researchers run a randomized block design using subject as the blocking variable. Assuming that there is no interaction effect, at the 2.5% significance level test the claim that at least one of the mean cholesterol levels for the five age groups is different from the others (Note: $\sum y_i^2 = 1,544,102$). Based on the results, can we conclude that 45 – 49 year olds probably have a higher average cholesterol level than people in their late twenties?

	Age Groups					
	25 - 29	30 - 34	35 - 39	40 - 44	45 – 49	**Totals**
Subject 1	185	277	294	300	310	1366
Subject 2	230	255	263	280	302	1330
Subject 3	165	167	170	171	180	853
Subject 4	303	335	360	385	420	1803
Total	883	1034	1087	1136	1212	**5352**

2. A restaurant chain wants to know if the brand of kitchen knife used by their chefs makes a difference in the amount of time it takes to prep before a dinner shift. They have four randomly chosen chefs complete a series of prep tasks using the different knives. Each chef will complete the prep work three different times each time using a different knife. The order of the knife usage will we randomized for each chef, so that familiarity with the task does not play a role. Assuming there is no interaction between chef and knife, at the 5% significance level test the claim that the three knives all have the same average completion time (Note: $\sum y_i^2 = 341.7174$). Based on the results, should the restaurant worry about which brand knife it chooses to purchase?

	Knife Brand			
	Knife A	Knife B	Knife C	**Total**
Chef 1	5.12	5.10	5.11	15.33
Chef 2	4.37	4.30	4.33	13
Chef 3	6.15	6.10	6.14	18.39
Chef 4	5.68	5.55	5.59	16.82
Total	21.32	21.05	21.17	**63.54**

3. A baker's yeast producer wants to know which ratio of cane molasses to beet molasses is ideal for producing a high yield of yeast. To test three different ratios, the company uses three different facilities, but to avoid differences between facilities from affecting the results, they run each ratio once through each facility in a random order. Complete the ANOVA table below and at a 1% significance level test if there is a difference between yeast yields using the different ratios of cane-to-beet molasses. Does the ratio of cane-to-beet molasses matter in the production process of yeast?

Source	df	SS	MS	F
Ratio		1.727		
Facility		0.107		
Error				
Total	8	1.84		

Answers:

1. $H_0 : \mu_{25} = \mu_{30} = \mu_{35} = \mu_{40} = \mu_{45}$

   H_a : at least one differs significantly from the others

Source	df	SS	MS	F
Age Group	4	15,268.3	3,817.075	7.413
Subject	3	90,459.6	30,153.2	58.560
Error	12	6,178.9	514.9083	
Total	19	111,906.8		

Critical value: $f_{4,12,0.025} = 4.1212$

$\text{Reject } H_0, \text{Support } H_a$

Since we rejected the null hypothesis and considering the sample data, it seems we can conclude that people in their late forties have higher average cholesterol rates than people in their late twenties.

2. $H_0 : \mu_A = \mu_B = \mu_C$

   H_a : at least one differs significantly from the others

Source	df	SS	MS	F
Knives	2	0.00915	0.004575	7.250
Chefs	3	5.26017	1.75339	2778.748
Error	6	0.00378	0.000631	
Total	11	5.2731		

Critical value: $f_{2,6,0.05} = 5.1433$

$\text{Reject } H_0, \text{Support } H_a$

Since we rejected the null hypothesis, the restaurant should pay attention to which knife it purchases for use in the restaurant. They are not all the same.

3. $H_0 : \mu_1 = \mu_2 = \mu_3$

H_a : at least one differs significantly from the others

Source	df	SS	MS	F
Ratio	2	1.727	0.8635	575.67
Facility	2	0.107	0.0535	35.67
Error	4	0.006	0.0015	
Total	8	1.84		

Critical value: $f_{2,4,0.01} = 18.00$

Since we rejected the null hypothesis, the ratio does seem to matter in the production of yeast.